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Ultraviolet fixed points in gauge and susy field theories in extra dimensions

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Abstract: We consider gauge field theories in $D > 4$ following the Wilson RG approach and show that they possess the ultraviolet fixed points where the gauge coupling is dimensionless in any space-time dimension. At the fixed point the anomalous dimensions of the field and vertex operators are known exactly. These fixed points are nonperturbative and correspond to conformal invariant theories. The same phenomenon also happens in supersymmetric theories with the Yukawa type interactions.

Keywords: Renormalization Group, Field Theories in Higher Dimensions.

Contents

1. Introduction

Nowadays it became popular to consider theories in extra dimensions as possible candidates for models of physics beyond the Standard Model. (See e.g. ref. [1, 2] and references therein.) One may wonder whether this extra dimensional theory can be considered as a consistent QFT in any sense. Since by general power counting it is nonrenormalizable, it looks hardly possible.

One way to consider an extra dimensional theory is the Kaluza-Klein approach. In this case, one takes the Fourier transform over the extra dimensions and obtains an infinite tower of states with quantized masses. Then one has to sum over all the states. This sum is usually divergent and a special prescription is needed to regularize it. Following this approach the divergences in $D = 5$ SUSY theory have been studied in [3, 4, 5] for the scalar effective potential. Some cancellations of UV divergences have been found. Doubtfully, however, that this approach solves the problem of nonrenormalizability in extra dimensions.

In principle, there is a chance that all the UV divergences cancel each other, like it takes place in $N = 4, 2$ and even $N = 1$ SUSY field theories in $D = 4$ [6], and one has a consistent theory. This possibility has been studied in the literature [7]–[10]. Though at lower orders the divergences indeed cancel on shell [7, 8, 10], in higher orders they may well appear being unprotected by any symmetry [9].

In what follows we first remind the situation with the UV divergences in SUSY gauge theories in extra dimensions in the lowest order and then discuss an alternative approach based on the Wilson renormalization group fixed points. The latter one is applied to the usual as well as supersymmetric gauge theories and exploits the nonperturbative RG fixed points for $D > 4$.

2. One-loop UV divergences in SUSY theories for arbitrary D

Consider the one-loop vacuum polarization diagram in a non-abelian gauge theory. It can be evaluated in arbitrary dimension using the technique of dimensional regularization. The result in the background field formalism is (we omit the transverse polarization tensor)

$$\begin{aligned}\Pi\left(p^2\right) &= (-)^{[D/2]}\frac{\Gamma(2-D/2)\Gamma^2(D/2)}{\Gamma(D)}\times\\ &\times\left\{-\left[2\frac{8(D-1)-D'}{D-2}+\frac{(D-4)(D-1)\alpha(8-\alpha)}{2(D-2)}+\frac{4}{D-2}\right]C_2(G)+\right.\\ &\left.+\,2^{[D'/2]}T(R)+\frac{4}{D-2}T(R)\right\}\frac{1}{(p^2)^{2-D/2}}\,,\end{aligned} \tag{2.1}$$

where D is the dimension of integration and D' is the dimension of the fields corresponding to the Lorentz algebra. We present the result in an arbitrary α-gauge ($\alpha = 0$ corresponds to the Feynman gauge). The square bracket contains the gauge and ghost field contribution, and then follows those of spinor and scalar fields.

Taking $D = 4-2\varepsilon$ in eq. (2.1) one can reproduce the result for the logarithmic, quartic and sextic divergences in $D = 4, 6$ and 10, respectively. The singular part is proportional to

$$-(26-D')C_2(G)+2^{[D'/2]}T(R)+2T(R)\,. \tag{2.2}$$

This is a gauge invariant expression of invariant operator $F^2_{\mu\nu}$.

Consider eq. (2.2) in particular cases corresponding to SUSY gauge theories in various dimensions taking the proper sets of the matter fields. The results are summarized below

$$\begin{array}{lll} D'=4 & N=1 & -22C_A+4C_A+4T_R+2T_R=-6(3C_A-T_R)\,,\\ & N=2 & -22C_A+4C_A+6C_A+12T_R=-12(C_A-T_R)\,,\\ & N=4 & -12C_A+12C_A=0\,,\\ D'=6 & N=1 & -20C_A+8C_A+8T_R+4T_R=-12(C_A-T_R)\,,\\ & N=2 & -12C_A+12C_A=0\,,\\ D'=10 & N=1 & -16C_A+16C_A=0\,. \end{array} \tag{2.3}$$

One can see that when the matter field representations are chosen in a proper way, the leading divergences indeed cancel each other. Note that the $N = 1$ $D = 10$ case coincides with the $N = 2$ $D = 6$ and $N = 4$ $D = 4$ ones and the $N = 1$ $D = 6$ case coincides with the $N = 2$ $D = 4$ one as expected.

Return to logarithmic divergences in higher dimensions. Take $D = 6$ for definiteness. Due to the background field gauge invariance the divergent structures in the one-loop order can take one of the following forms:

$$\begin{aligned} I_1 &= TrD_\rho F_{\mu\nu}D_\rho F_{\mu\nu}\,,\\ I_2 &= TrD_\mu F_{\mu\nu}D_\rho F_{\rho\nu}\,,\\ I_3 &= TrD_\rho F_{\mu\nu}D_\mu F_{\rho\nu}\,,\\ I_4 &= TrF_{\mu\nu}F_{\nu\rho}F_{\rho\mu}\,. \end{aligned} \tag{2.4}$$

However, these invariants are not independent. Due to the relation $[D_\mu, D_\nu] = F_{\mu\nu}$ and the Bianchy identity $D_\mu F_{\nu\rho} + D_\rho F_{\mu\nu} + D_\nu F_{\rho\mu} = 0$, one has only 2 independent structures and can choose any of them. We take the first two. Then calculating the diagrams and extracting the contribution to two independent Lorentz structures one can find the coefficients of them. The result is

$$\frac{T_R - C_A}{3}\, Tr D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}\,. \tag{2.5}$$

One finds that the result for ALL the structures is proportional to $\sum T(R) - C_2(G)$, like for the quadratic divergences, and vanishes *off-shell*. Due to the fact that all the structures vanish we claim that all the one loop divergences in the gauge sector cancel for $\sum T(R) = C_2(G)$!

However, unlike the quadratic divergences, this result is gauge-dependent. In an arbitrary α-gauge eq. (2.5) looks like

$$\frac{T_R - C_A(1+\alpha-\alpha^2/8)}{3}\, Tr D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}\,. \tag{2.6}$$

and the cancellation is not obvious anymore.

To get a gauge invariant statement, one has to go on-shell, i.e. to use the equations of motion. For the pure gauge case they are

$$D_\mu F_{\mu\nu} = \bar\lambda \gamma^\nu \lambda\,, \qquad \hat{D}\lambda = 0\,, \tag{2.7}$$

where λ is the gaugino field. Collecting the terms of effective action which transform into one another due to the equations of motion one has

$$\ldots (D_\mu F_{\mu\nu})^2 + \ldots \bar\lambda\gamma^\nu D_\mu F_{\mu\nu}\lambda + \ldots \left(\bar\lambda\gamma^\nu\lambda\right)^2 = 0 \quad !\,,$$

where the dots stand for the know coefficients. That is one finds cancellation of the logarithmic divergences *on-shell* in any gauge.

In higher loops the following statements are valid:

1. The on-shell finiteness of the $D = 6$ $N = 1$ SUSY gauge theory is true in two loops as well. This has been checked by explicit calculation in components [7, 8];

2. Within the (constrained) superfield formalism it is possible to show that the allowed invariants vanish on-shell up to 2 loops. However, in higher loops the nonvanishing invariants exist [9]. The coefficients are not calculated but there is no known symmetry that might protect them.

Thus, our main conclusion is not optimistic: there is no big chance for the cancellation of logarithmic divergences for $D > 4$ even on-shell, i.e. the theory remains *perturbatively nonrenormalizable*.

3. Nonperturbative fixed point in gauge theories for $D > 4$

We turn now to an alternative idea and look for nonperturbative possibilities to construct a viable higher dimensional theory. We follow the so-called Wilson Renormalization Group approach [11], only not in a scalar theory but in a gauge one. Our treatment of nonrenormalizable interactions follows that of M.Strassler [12, 13].

Consider first the usual gauge theory in D dimensions

$$\mathcal{L} = -\frac{1}{4} Tr F_{\mu\nu}^2 , \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] . \tag{3.1}$$

The fields and the coupling have the following canonical dimensions:

$$[A] = \frac{D-2}{2} , \qquad [F] = \frac{D}{2} , \qquad [g] = 2 - \frac{D}{2} .$$

This means that $D = 4$ is the critical dimension for the gauge interaction: the coupling here is dimensionless, the operators are marginal and the theory is renormalizable in a usual sense.

To go beyond the critical dimension we follow the standard approach [14] (see also ref. [15]) based on dimensional regularization and analytical continuation. Consider the dimensionless quantity

$$\tilde{g} \equiv g\mu^{D/2-2} \qquad \Rightarrow \qquad [\tilde{g}] = 0 ,$$

where μ is some scale, and expresses the bare coupling in terms of a renormalized one

$$g_B = \mu^{2-D/2} \tilde{g} Z_g(\tilde{g}) , \tag{3.2}$$

where the renormalization constant Z_g depends on D and in the minimal subtraction scheme contains only the pole terms in $(D - D_c)$ with $D_c = 4$ in this case.

The crucial point here is that the renormalization constant depends on $\tilde{g}$ and does not contain the infinite number of higher dimensional operators that may appear at $D > D_c$. In Wilson approach these operators are irrelevant while going toward the infrared direction and may be ignored [11]. Remind, however, that in this case we intend to describe the IR properties of the model which are universal and do not depend on the UV details. The latter happen to be out of control due to severe UV divergences.

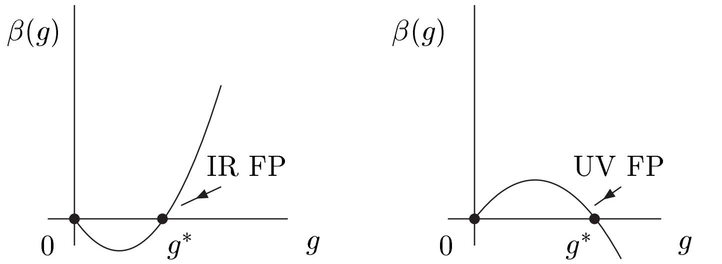
Differentiating then eq. (3.2) with respect to a scale keeping g_b fixed one gets

$$0 = \left(2 - \frac{D}{2}\right) \tilde{g} + \beta(\tilde{g}) - \tilde{g}\gamma_g(\tilde{g}) , \tag{3.3}$$

where as usual $\beta(\tilde{g}) = \mu \frac{d}{d\mu} \tilde{g}|_{g_b}$ and $\gamma_g = -\mu \frac{d}{d\mu} ln Z_g|_{g_b}$. This leads to the following RG equation for the coupling

$$\mu \frac{d}{d\mu} \tilde{g} = \beta(\tilde{g}) = \tilde{g} \left(\frac{D}{2} - 2 + \gamma_g\right) . \tag{3.4}$$

In general $\gamma_g(g)$, and hence $\beta(g)$, may depend on D being finite while $D \to D_c$. However, in the MS-scheme this dependence is absent and γ_g can be calculated directly in the critical



Scalar Theory ($D < 4$) Gauge Theory ($D > 4$)

Figure 1: The β functions for scalar and gauge field theories in D-dimensions.

dimension. We use this advantage since while the β function and the anomalous dimensions are scheme dependent, the value of the anomalous dimension at the critical point is universal. This means that it can be calculated in any scheme.

In what follows it is useful to proceed in the background field gauge where $\gamma_g = \frac{1}{2}\gamma_A$ the latter being the gauge field anomalous dimension.

Consider now the RG equation (3.4) in an arbitrary dimension D not necessarily close to 4. It has a fixed point. In fact, two of them

$$\begin{aligned} &1) \quad \tilde{g} = 0 \quad \rightarrow g = 0\,, \quad \gamma_A = 0\,, \\ &2) \quad \tilde{g} = g^*\,, \quad \gamma_A = 4 - D\,. \end{aligned}$$

The first one is trivial, this is the so-called gaussian fixed point. It is perturbative. The second one is nonperturbative, it is the so-called Wilson-Fisher fixed point [11]. The anomalous dimension here is not small, it is integer. It is achieved at the value of the coupling which is unknown, though the value of the anomalous dimension is known *exactly*. Since the anomalous dimension in gauge theories, contrary to the scalar case, is negative, the fixed point of the second kind exists for $D > 4$. Remind that in scalar theories it exists for $D < 4$: one takes $D = 4 - \epsilon$, where $\epsilon \to 1$ or 2 and performs the so-called $\epsilon-$expansion [11]. In the case of a scalar theory the FP is IR stable, while in a gauge theory it is UV stable (see figure 1).

Such a fixed point in a gauge theory within the ϵ-expansion has been advocated in ref. [16]. Some additional supporting arguments in favour of the fixed point in 5 dimensions come from the lattice calculations [17]. At last, there is also very useful analogy between the gauge theory and the nonlinear sigma-model. The latter has a critical dimension equal to 2 and is asymptotically free there. One can go above the critical dimension with the help of the $2 + \epsilon$ expansion. Then the theory has a fixed point in the leading order [18] which is also true within the 1/N expansion performed directly in three dimensions [19].

Consider the properties of the fixed point #2. Let us calculate the dimensions. One has for the field

$$[A] = \frac{D-2}{2} + \frac{1}{2}\gamma_A = \frac{D-2}{2} + \frac{4-D}{2} = 1$$

in any D. To calculate the dimension of the coupling, one has to consider the vertex $g\partial A[A,A]$ which gives

$$D = [g] + 1 + 3[A] + \gamma_V \,.$$

Since $\gamma_V = -\gamma_A$ in the background gauge, one obtains

$$[g] = D - 4 - \gamma_V = D - 4 + \gamma_A = 0 \qquad \text{in any D} \quad !$$

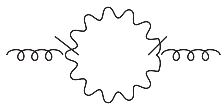
Thus, one has a dimensionless coupling at the fixed point that means renormalizability. *The theory at the fixed point is perturbatively nonrenormalizable, but nonperturbatively renormalizable!* (cf ref. [13]). The existence of a renormalizable field theory beyond PT relies, in the sense of statistical physics, on the existence of a fixed point (see e.g ref. [15], p.549).

How can one understand this statement in terms of Feynman diagrams? Compare the two fixed points, the gaussian one and the nonperturbative one

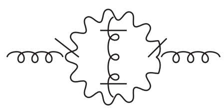
$g = 0$	$g = g^*$
$\widehat{AA} \sim \dfrac{1}{(x^2)^{\frac{D-2}{2}}}$	$\widehat{AA} \sim \dfrac{1}{(x^2)^1}$
$\int \dfrac{d^D x e^{ipx}}{(x^2)^{\frac{D-2}{2}}} \sim \dfrac{1}{p^2}$	$\int \dfrac{d^D x e^{ipx}}{(x^2)} \sim \dfrac{1}{(p^2)^{\frac{D-2}{2}}}$

Thus, for instance, for $D = 6$ at the non-gaussian fixed point the propagator behaves like $1/p^4$, i.e. much faster than in the usual case.

One can consider the diagrams with modified Feynman rules taking into account the anomalous dimensions. This corresponds to infinite summation of subgraphs. For the gauge propagator one has by power counting



$$\frac{D + 2 - 2 + 2\gamma_V}{4 - 2\gamma_A} \Rightarrow D - 4 \,,$$



$$\frac{2D + 4 - 2 + 4\gamma_V}{10 - 5\gamma_A} \Rightarrow 2D - 8 + \gamma_A = D - 4 \,,$$

. . . .

$$\Rightarrow \quad D - 4 \,.$$

Hence, one has the same power in any loop, that is renormalizability. This is the consequence of dimensionless coupling at the fixed point.

One can try to construct an effective lagrangian that describes these diagrams. In $D = 6$, as it is suggested by the one-loop calculation (2.5) and the behaviour of the propagator, it may be

$$\mathcal{L}_{\text{eff}} \sim \; Tr(D_\mu F_{\mu\nu})^2 \,. \tag{3.5}$$

The effective lagrangian (3.5) has some remarkable properties

- It has no scale, the coupling is dimensionless;
- It is scale (conformal) invariant;
- The exact anomalous dimensions of the field and vertices are taken into account;
- It is vanishing on-shell ($D_\mu F_{\mu\nu} = 0$).

At first sight, the effective lagrangian (3.5) contains higher derivatives, and hence, ghosts. However, it is an effective theory and it is not clear for us how to define the spectrum of effective theory: is it the spectrum of the original lagrangian or may be some new fields are adequate in this case?

4. Nonperturbative fixed point in SUSY theories for $D > 4$

A similar phenomenon takes place in SUSY gauge theories. Again we start at the critical dimension $D = 4$ and use $N = 1$ superfields. Strictly speaking, they are $D = 4$ superfields; however, component notation is more cumbersome and what we really need are the renormalizations in a critical dimension. So the superfield formalism here is not rigorous but useful. Remind also that supersymmetry is possible only at integer dimensions with $D \leq 10$ if one restricts the maximum spin=1.

The SUSY lagrangian looks like (we omit the gauge fields for the moment)

$$\mathcal{L} = \int d^4\theta \; \bar{\Phi}_i \Phi_i + \int d^2\theta \; \mathcal{W} + h.c. \,, \qquad \mathcal{W} = y \Phi_1 \Phi_2 \Phi_3 \,. \tag{4.1}$$

Calculating the dimensions of the fields and the Yukawa coupling y, one has

$$\begin{aligned} &[\mathcal{L}] = D \,, \qquad [d\theta] = \frac{1}{2} \,, \qquad [\mathcal{W}] = D - 1 \,, \\ &[\Phi] = \frac{D-2}{2} \,, \qquad [y] = D - 1 - 3\frac{D-2}{2} = 2 - \frac{D}{2} \,. \end{aligned}$$

Now we proceed as above. Introduce a dimensionless quantity $\tilde{y} = y\mu^{D/2-2}$ and write the RG equation for y in $D = 4$

$$\mu \frac{d}{d\mu} y = y \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\gamma_2 + \frac{1}{2}\gamma_3 \right) , \tag{4.2}$$

where γ_i is the anomalous dimension of the matter field Φ_i. We use here the nonrenormalization theorem in $D = 4$ which states that the anomalous dimension of the vertex is zero.

This allows us to get the RG equation for $\tilde{y}$

$$\mu \frac{d}{d\mu} \tilde{y} = \tilde{y} \left(\frac{1}{2}\gamma_1 + \frac{1}{2}\gamma_2 + \frac{1}{2}\gamma_3 \right) + \tilde{y} \left(\frac{D}{2} - 2 \right) = \frac{\tilde{y}}{2}(\gamma_1 + \gamma_2 + \gamma_3 + D - 4). \tag{4.3}$$

This equation has two fixed points [12][1]

$$1)\quad \tilde{y}=0 \quad \to y=0\,, \quad \gamma_i=0\,,$$
$$2)\quad y=y^*\,, \quad \gamma_i=\frac{4-D}{3}\,.$$

One can see that for $D>4$ the second fixed point requires the anomalous dimension to be negative. This is only possible in gauge theories. Hence, in fact one has to consider the gauge invariant SUSY theory where the nontrivial fixed point is (g^*, y^*). At this point the dimension of the Yukawa coupling is

$$[y^*] = D-1-3\frac{D-2}{2}-\frac{\gamma_1+\gamma_2+\gamma_3}{2}=0 \qquad \text{in any D} \quad !$$

Thus, again, we get a theory that is perturbatively nonrenormalizable, but nonperturbatively renormalizable at the nontrivial fixed point. At this point a theory possess all the properties mentioned above.

There is, however, one subtlety with supersymmetry. While supersymmetric gauge theory exists for $D \leq 10$, it is known that superconformal algebra, which we assume to be realized at the fixed point, is only possible for $D \leq 6$. This is due to the classification of all possible superconformal algebras given in ref. [22]. At the same time due to the vanishing beta function the conformal anomaly should also vanish at the fixed point so the theory is scale invariant. Hence, we find some kind of discrepancy here[2]. There might be two ways out of it: either the fixed point solution is not valid for some reason above $D=6$, or the full superconformal algebra is not realized at the fixed point. This remains an open question so far.

One may wonder whether this nontrivial fixed point is reachable. To see this, consider an $N=1$ SUSY gauge theory and take the all-loop NSVZ β-function [21]. Extracting the γ_A one finds

$$\gamma_A = 2\alpha\frac{T_R-3C_A-\frac{2}{r}\sum C_R\gamma_R}{1-2C_A\alpha}\,,$$

where $\alpha \equiv g^2/16\pi^2$.

For a pure SUSY Yang-Mills case one has the equation

$$\gamma_A = 2\alpha\frac{-3C_A}{1-2C_A\alpha} = 4-D\,.$$

The solution is

$$\alpha^* = \frac{D-4}{D-1}\frac{1}{2C_A}\,.$$

This value is smaller than the pole value $\alpha_{pole}=1/2C_A$. In particular, in $D=6$ one has $\alpha^*=1/5C_A$, as shown in figure 2. Thus, the fixed point seems to be quite reachable.

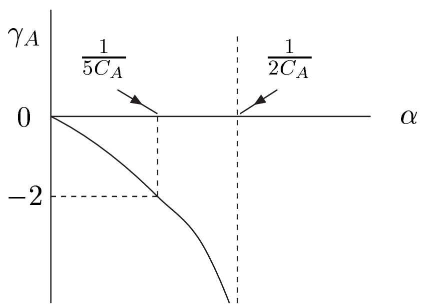


Figure 2: The gauge field anomalous dimension in SUSY field theory.

[1]For a scalar SUSY theory in $D<4$ this nonperturbative fixed point was earlier used in ref. [20] to describe the self-avoiding random walk.

[2]I am grateful to E.Witten for pointing out this problem to me.

5. Conclusion

Summarizing the analysis of the gauge and SUSY field theories in higher dimensions from the point of view of their renormalizability and consistency, we come to the following conclusions

- Perturbative finiteness in $D > 4$ seems not to be valid;
- Within the Wilson RG approach the nontrivial nonperturbative fixed points may lead to nonperturbative renormalizability;
- These theories may be related to PT renormalizable effective models which have to be found;
- At the fixed point the theory possesses the conformal invariance, and the anomalous dimensions are known exactly.

These observed fixed points may be related to those originated from the string dynamics known already for several years for $D = 5$ and 6 (see ref. [23] and references therein). We use here the more familiar language that is close to statistical physics and critical phenomena. In a sense we give an explicit example of a local field theory with non-trivial fixed points thus strengthening the claim (based on string theory) that exist field theories that flow to non-trivial fixed points in more than 3 dimensions [23]. It would be very useful to our mind to find explicit link between the two approaches.

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